

A Practical Guide to
Quantitative Finance
Interviews



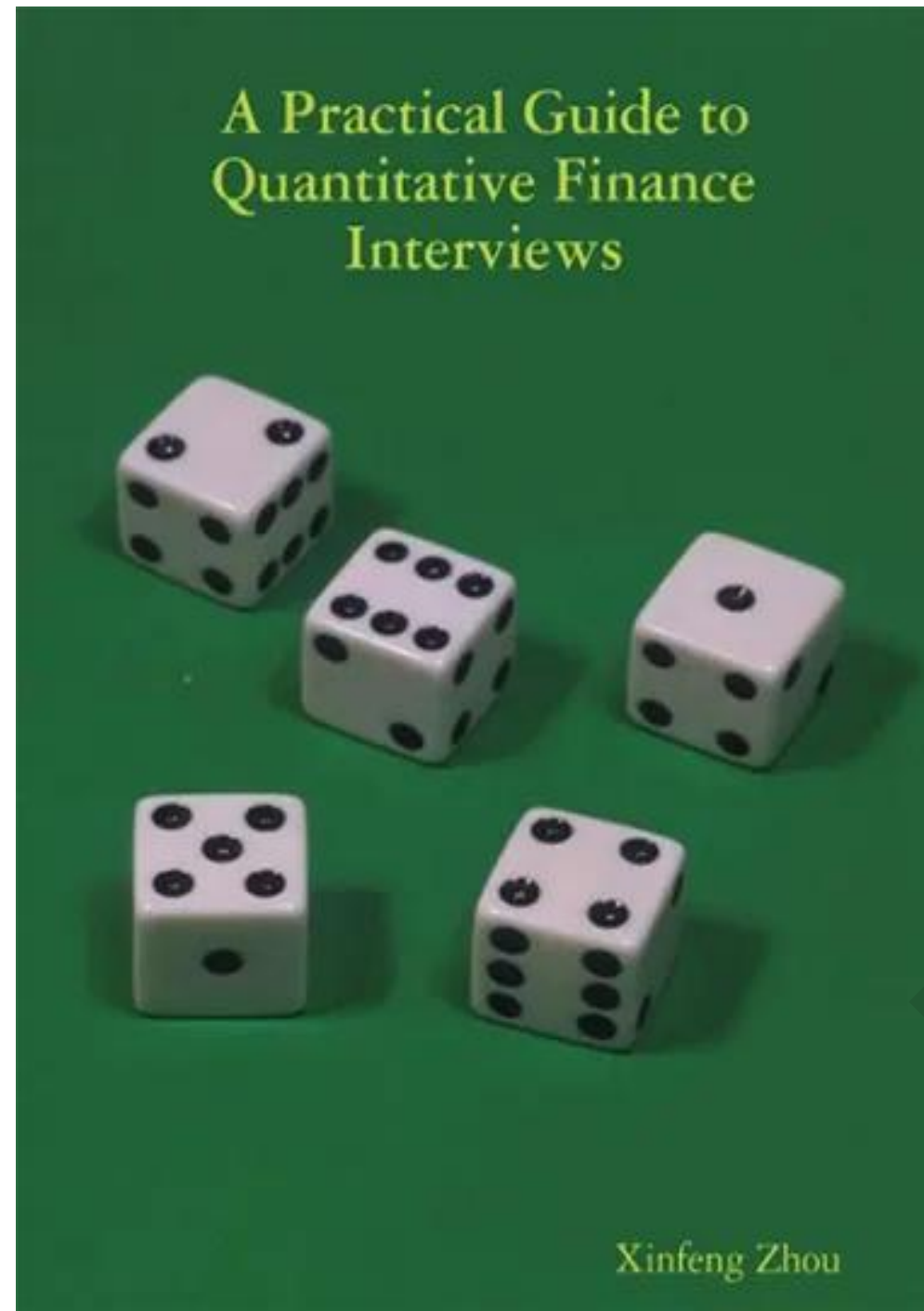
Xinfeng Zhou

绿皮书精讲60题

Quant Interview

60 Questions

为什么刷绿皮书



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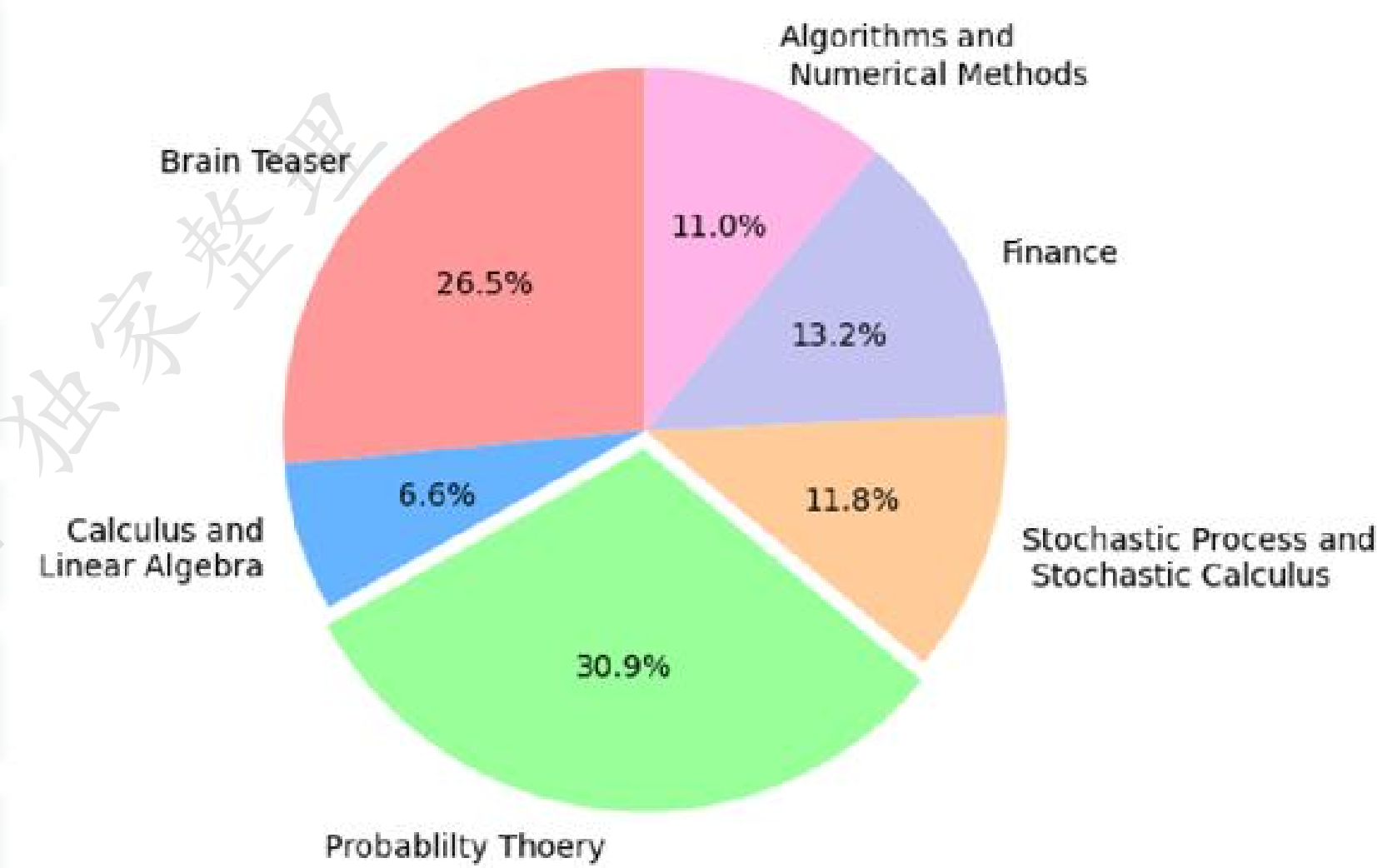
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- 2008出版，距今已经过去15年，但仍为经典
- 题目难度较大，做过就会，没做过基本不会
- 名校学生太多，总要设置一些门槛
- 题目涵盖统计、概率、计算机、金融、数学
- 这些题能刷完，代表智商在线

绿皮书题目分布

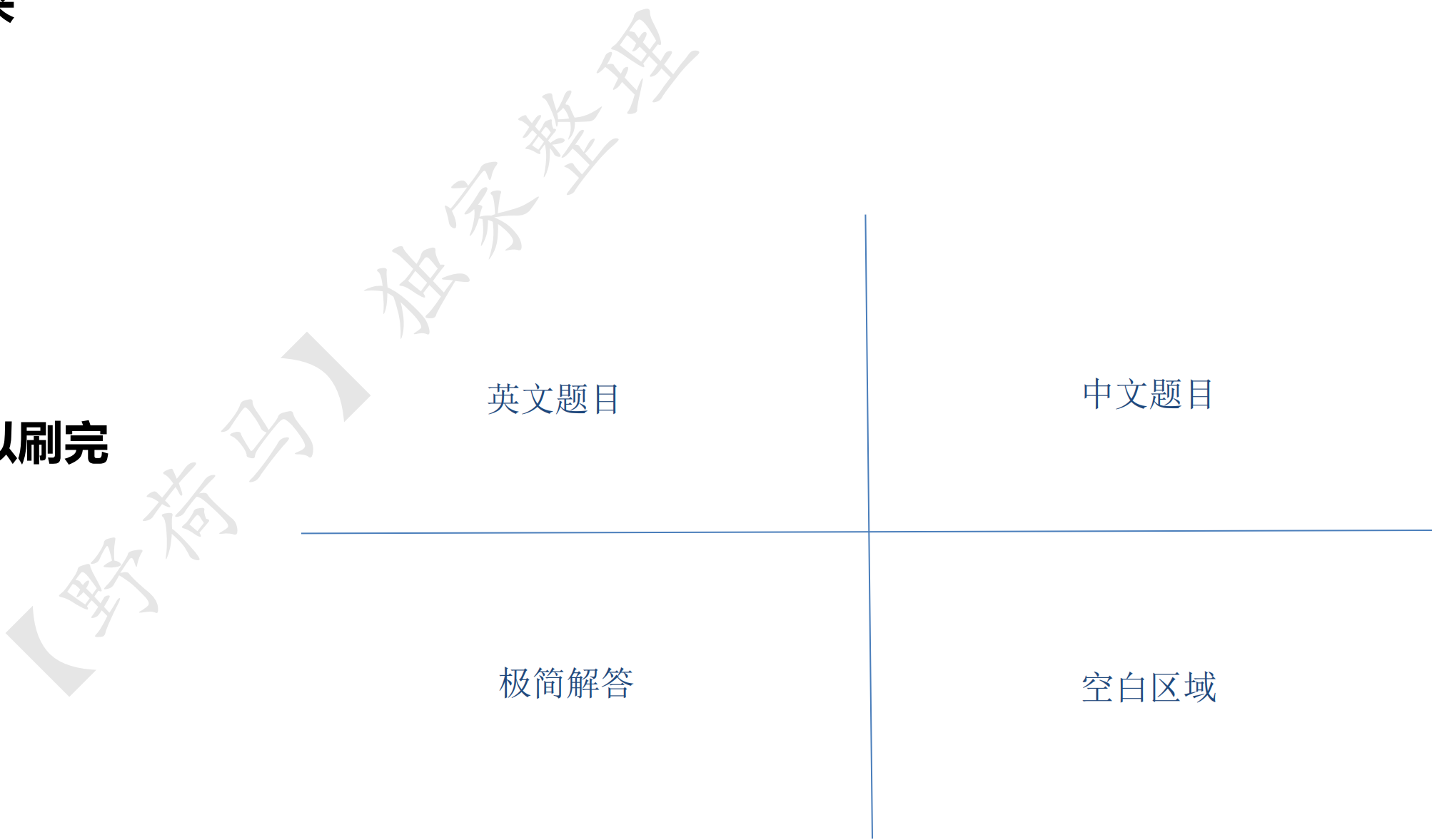
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绿皮书刷题方法

- 快速过一遍，把不会的题标出来
- 过第二遍，重点击破不会的题
- 整体再做第三遍，查漏补缺

建议每天1~2小时，一般四周可以刷完



Q1: Screwy privates 海盜分金

Five pirates looted a chest full of 100 gold coins. Being a bunch of democratic pirates, they agree on the following method to divide the loot:

The most senior pirate will propose a distribution of the coins. All pirates, *including the most senior pirate*, will then vote. If at least 50% of the pirates (3 pirates in this case) accept the proposal, the gold is divided as proposed. If not, the most senior pirate will be fed to shark and the process starts over with the next most senior pirate... The process is repeated until a plan is approved. You can assume that all pirates are perfectly rational: they want to stay alive first and to get as much gold as possible second. Finally, being blood-thirsty pirates, they want to have fewer pirates on the boat if given a choice between otherwise equal outcomes.

How will the gold coins be divided in the end?

五个海盜抢走了一个装满 100 枚金币的箱子。作为一群民主的海盜，他们同意以下分配战利品的方法：

最资深的海盜将提议分配硬币。所有的海盜，包括最资深的海盜，都会进行投票。如果至少 50% 的海盜（在本例中为 3 名海盜）接受提议，则黄金按提议进行分配。如果没有，则将最高级的海盜喂给鲨鱼，然后从下一个最高级的海盜开始这个过程..... 重复这个过程，直到一个计划被批准。你可以假设所有的海盜都是完全理性的：他们首先想活下去，其次是获得尽可能多的金子。最后，作为嗜血的海盜，如果在其他方面均等的结果之间做出选择，他们希望船上的海盜更少。

金币最终将如何分配？

Screwpy pirates

原题地址：Page 3

Solution=

	A	B	C	D	E
1 pirate					100
2 pirates				0	100
3 pirates			1	0	99
4 pirates		0	1	0	99
5 pirates	1	0	1	0	98

Q2: Tiger and sheep 老虎和羊

One hundred tigers and one sheep are put on a magic island that only has grass. Tigers can eat grass, but they would rather eat sheep. Assume: *A.* Each time only one tiger can eat one sheep, and that tiger itself will become a sheep after it eats the sheep. *B.* All tigers are smart and perfectly rational and they want to survive. So will the sheep be eaten?

一百只老虎和一只羊被放在一个只有草的魔法岛上。老虎吃草，但他们宁愿吃羊。
假设： **A.** 每次只有一只老虎可以吃一只羊，而这只老虎吃完羊后自己也会变成羊。
B. 所有的老虎都很聪明，而且非常理性，它们都想生存。
那么羊会被吃掉吗？

原题地址： [Page 4](#)

Tiger and sheep

● Solution= 递归

Tiger	sheep	
1	1	eat
2	1	not eat
3	1	eat
4	1	not eat
...
100	1	not eat

Q3: River crossing 过河

Four people, A , B , C and D need to get across a river. The only way to cross the river is by an old bridge, which holds at most 2 people at a time. Being dark, they can't cross the bridge without a torch, of which they only have one. So each pair can only walk at the speed of the slower person. They need to get all of them across to the other side as quickly as possible. A is the slowest and takes 10 minutes to cross; B takes 5 minutes; C takes 2 minutes; and D takes 1 minute.

What is the minimum time to get all of them across to the other side?¹

A、B、C、D 四个人需要过河。

过河的唯一方法是通过一座旧桥，一次最多可容纳 2 人。

由于天黑，他们没有手电筒就无法过桥，而他们只有一个手电筒。

所以每一对只能以较慢者的速度行走。他们需要尽快让所有人都渡过对岸。A 最慢，需要 10 分钟才能通过；B 用时 5 分钟；C 需要 2 分钟；D 需要 1 分钟。

让他们全部渡过对岸的最短时间是多少？

原题地址：Page 5

River crossing

● Solution = A 10 min B 5 min C 2 min D 1 min

→ $C+D = 2$ min ← D 1 min

→ $A+B = 10$ min ← C 2 min

→ $C+D = 2$ min

= 17 min

Q4: Birthday problem 生日问题

You and your colleagues know that your boss *A*'s birthday is one of the following 10 dates:

Mar 4, Mar 5, Mar 8

Jun 4, Jun 7

Sep 1, Sep 5

Dec 1, Dec 2, Dec 8

A told you only the month of his birthday, and told your colleague *C* only the day. After that, you first said: "I don't know *A*'s birthday; *C* doesn't know it either." After hearing what you said, *C* replied: "I didn't know *A*'s birthday, but now I know it." You smiled and said: "Now I know it, too." After looking at the 10 dates and hearing your comments, your administrative assistant wrote down *A*'s birthday without asking any questions. So what did the assistant write?

Birthday problem

● Solution= 3.4 3.5 3.8

6.4 6.7

9.1 9.5

12.1 12.2 12.8

①	I ×	C ×	6.12月	No
②	I ×	C ✓	3.5 9.5	No
③	I ✓	C ✓	9.1	Yes

你和你的同事知道你的老板 *A* 的生日是以下 10 个日期之一：

3 月 4 日、3 月 5 日、3 月 8 日

6 月 4 日、6 月 7 日

9 月 1 日、9 月 5 日

12 月 1 日、12 月 2 日、12 月 8 日

A 只告诉你他生日的月份，而只告诉你的同事 *C* 生日那天。

之后，你先说：“我不知道 *A* 的生日，*C* 也不知道。”

听完你的话，*C* 回答说：“以前不知道 *A* 的生日，现在知道了。”

你笑着说：“现在我也知道了。” 在查看了 10 个日期并听取了您的意见后，您的行政助理在没有询问任何问题的情况下写下了 *A* 的生日。

那么助理写了什么？

原题地址：Page 5

Q5: Card game 扑克牌游戏

A casino offers a card game using a normal deck of 52 cards. The rule is that you turn over two cards each time. For each pair, if both are black, they go to the dealer's pile; if both are red, they go to your pile; if one black and one red, they are discarded. The process is repeated until you two go through all 52 cards. If you have more cards in your pile, you win \$100; otherwise (including ties) you get nothing. The casino allows you to negotiate the price you want to pay for the game. How much would you be willing to pay to play this game?²

赌场提供使用一副普通的 52 张牌的纸牌游戏。规则是每次翻两张牌。对于每一对，如果都是黑色，则它们进入庄家堆；如果两者都是红色的，它们就会进入你的堆；如果一黑一红，则丢弃。重复该过程，直到你们两个完成所有 52 张卡片。如果您的牌堆中有更多牌，您将赢得 100 美元；否则（包括关系）你什么也得不到。赌场允许您协商要为游戏支付的价格。你愿意花多少钱玩这个游戏？

原题地址：Page 6

Card game

● Solution=

黑 红 N

黑 26-N

红 26-N

Always equal

Q6: Burning ropes 烧绳子

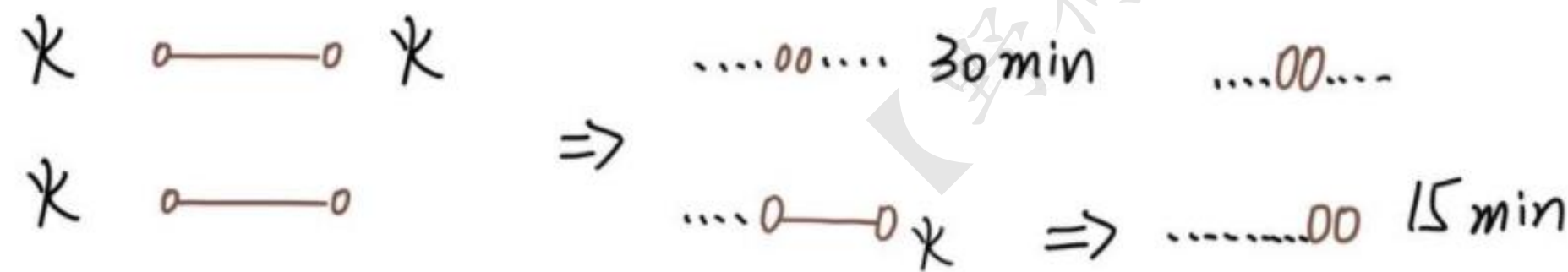
You have two ropes, each of which takes 1 hour to burn. But either rope has different densities at different points, so there's no guarantee of consistency in the time it takes different sections within the rope to burn. How do you use these two ropes to measure 45 minutes?

你有两根绳子，每根绳子燃烧需要 1 小时。但是任何一根绳子在不同点都有不同的密度，所以不能保证绳子内不同部分燃烧的时间的一致性。
你如何用这两条绳子来测量 45 分钟？

原题地址: Page 7

Burning ropes

● Solution=



Q7: Trailing zeros 结尾零

How many trailing zeros are there in 100! (factorial of 100)? 100 中有多少个尾随零! (100 的阶乘) ?

原题地址: Page 9

Trailing zeros

● Solution= 100!

5的倍数 5 10 15 20 100 20个

25的倍数 25 50 75 100 4个

2的倍数 50个

故 24个0

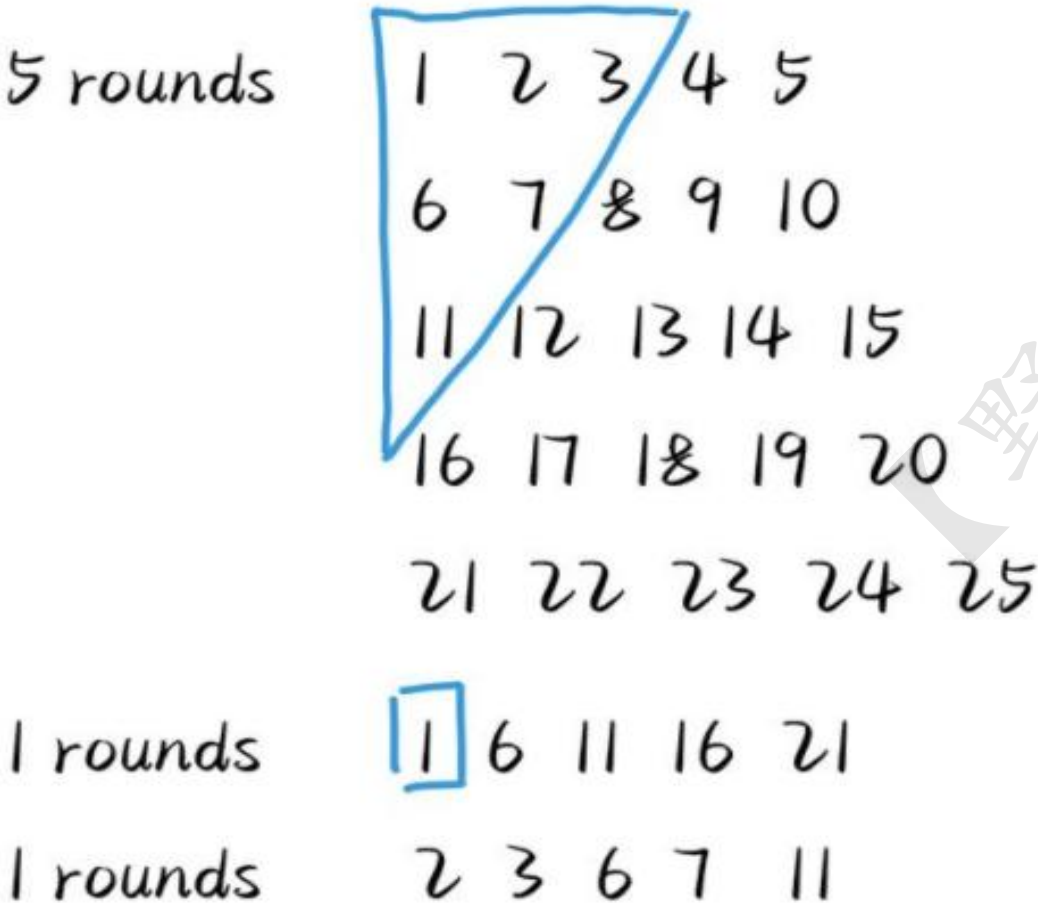
Q8: Horse race 赛马比赛

There are 25 horses, each of which runs at a constant speed that is different from the other horses'. Since the track only has 5 lanes, each race can have at most 5 horses. If you need to find the 3 fastest horses, what is the minimum number of races needed to identify them?

有25匹马，每匹马都以不同于其他马匹的恒定速度奔跑。由于赛道只有 5 条车道，每场比赛最多可以有 5 匹马。
如果需要找出最快的 3 匹马，最少需要多少场比赛才能确定？

原题地址：Page 9

Horse race



Q9: Door to offer 通往offer的门

You are facing two doors. One leads to your job offer and the other leads to exit. In front of either door is a guard. One guard always tells lies and the other always tells the truth. You can only ask one guard one yes/no question. Assuming you do want to get the job offer, what question will you ask?

你面对着两扇门。一个通往你的工作机会，另一个是退出。两扇门前都有一名警卫。一个守卫总是说谎，另一个总是说真话。您只能问一名警卫一个是/否问题。

假设你确实想获得工作机会，你会问什么问题？

原题地址：Page 12

Door to offer

Solution=

What the other guard say that you are guarding the door to the offer?

Yes. The other door

No. The door in front of you

Q10: Message delivery 消息传递

You need to communicate with your colleague in Greenwich via a messenger service. Your documents are sent in a padlock box. Unfortunately the messenger service is not secure, so anything inside an unlocked box will be lost (including any locks you place inside the box) during the delivery. The high-security padlocks you and your colleague each use have only one key which the person placing the lock owns. How can you securely send a document to your colleague?⁶

您需要通过信使服务与您在格林威治的同事进行交流。您的文件将在一个带锁的盒子中发送。不幸的是，**Messenger** 服务并不安全，因此未上锁的盒子内的任何物品（包括您放置在盒子内的任何锁）都将在递送过程中丢失。您和您的同事每次使用的高安全性挂锁只有一把钥匙，钥匙由开锁者拥有。
您如何安全地将文件发送给您的同事？

原题地址：Page 13

Message delivery

=> A —  — B

=> A —   — B

=> A —  — B

Q11: Last ball 最后一个球

A bag has 20 blue balls and 14 red balls. Each time you randomly take two balls out. (Assume each ball in the bag has equal probability of being taken). You do not put these two balls back. Instead, if both balls have the same color, you add a blue ball to the bag; if they have different colors, you add a red ball to the bag. Assume that you have an unlimited supply of blue and red balls, if you keep on repeating this process, what will be the color of the last ball left in the bag?⁷ What if the bag has 20 blue balls and 13 red balls instead?

一个袋子里有 20 个蓝球和 14 个红球。每次随机取出两个球。（假设袋中每个球被取出的概率相等）。你不要把这两个球放回去。相反，如果两个球的颜色相同，则向袋子中添加一个蓝色球；如果它们有不同的颜色，你就在袋子里加一个红球。假设你有无限量的蓝球和红球，如果你一直重复这个过程，袋子里剩下的最后一个球是什么颜色？如果袋子里有 20 个蓝球和 13 个红球会怎样？

原题地址：Page 13

Last ball

2 red

$$(B, R) \rightarrow (B+1, R-2)$$

2 blue

$$(B, R) \rightarrow (B-1, R)$$

1 red 1 blue

$$(B, R) \rightarrow (B-1, R)$$

R都少2个

Q12: Quant salary 量化工资

Eight quants from different banks are getting together for drinks. They are all interested in knowing the average salary of the group. Nevertheless, being cautious and humble individuals, everyone prefers not to disclose his or her own salary to the group. Can you come up with a strategy for the quants to calculate the average salary without knowing other people's salaries?
securely send a document to your colleague?

来自不同银行的八位量化分析师聚在一起喝酒。他们都很想知道该组的平均工资。

然而，作为谨慎和谦虚的人，每个人都不愿意向团队透露自己的薪水。

你能想出一个让量化分析师在不知道其他人工资的情况下计算平均工资的策略吗？

原题地址：Page 15

Quant salary

Random x

$$\sum_{i=1}^8 y_i + x$$

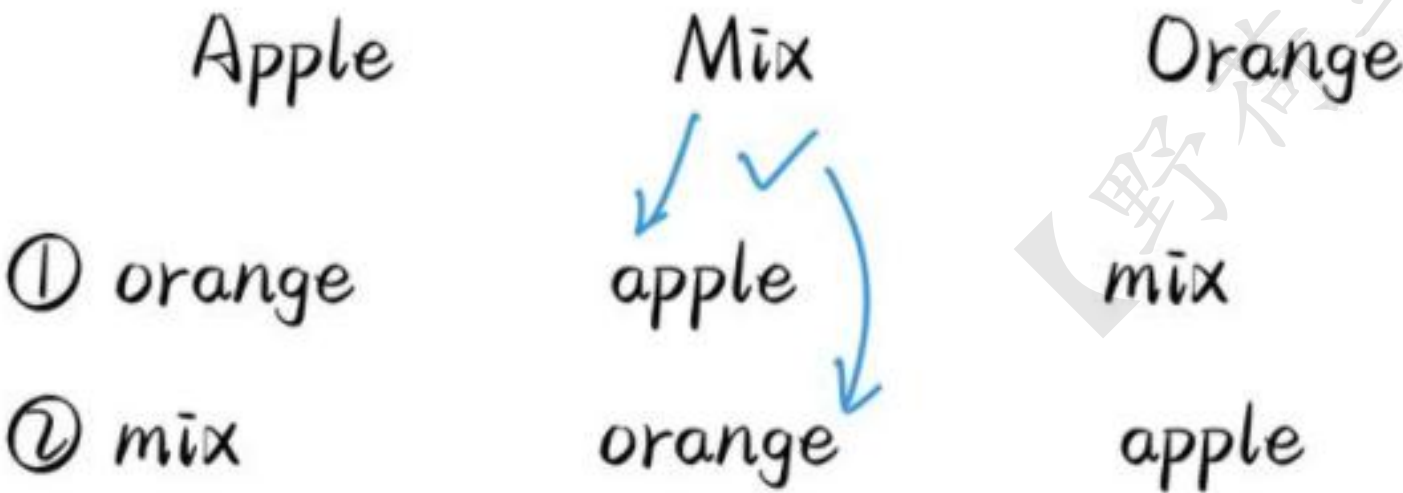
Q13: Mislabeled bags 贴错标签的袋子

You are given three bags of fruits. One has apples in it; one has oranges in it; and one has a mix of apples and oranges in it. Each bag has a label on it (apple, orange or mix). Unfortunately, your manager tells you that ALL bags are mislabeled. Develop a strategy to identify the bags by taking out minimum number of fruits? You can take any number of fruits from any bags.⁸

给你三袋水果。一个里面有苹果；一个里面有橘子；其中一个混合了苹果和橙子。每个袋子上都有一个标签（苹果、橙子或混合）。不幸的是，您的经理告诉您所有行李都贴错了标签。制定一种策略，通过取出最少数量的水果来识别袋子？您可以从任何袋子中取出任意数量的水果。

原题地址：Page 16

Mislabeled bags



Q14: Clock pieces 钟表零件

A clock (numbered 1 – 12 clockwise) fell off the wall and broke into three pieces. You find that the sums of the numbers on each piece are equal. What are the numbers on each piece? (No strange-shaped piece is allowed.)

一个时钟（顺时针编号为 1-12）从墙上掉了下来，摔成三块。
你发现每块上的数字之和是相等的。
每块上的数字是多少？（不允许出现异形件。）

原题地址：Page 18

Clock piece

$$\sum_{i=1}^{12} i = 78$$

$$78 / 3 = 26$$

- ① 5 6 7 8 13
- ② 11 12 12
- ③ 3 4 9 10

Q15: Counterfeit coins I 假币一

There are 10 bags with 100 identical coins in each bag. In all bags but one, each coin weighs 10 grams. However, all the coins in the counterfeit bag weigh either 9 or 11 grams. Can you find the counterfeit bag in only one weighing, using a digital scale that tells the exact weight?⁹

有 10 个袋子，每个袋子里有 100 个相同的硬币。在除一个以外的所有袋子中，每枚硬币重 10 克。然而，假币袋中的所有硬币重达 9 克或 11 克。您是否可以使用显示准确重量的数字秤，仅通过一次称重就找到假冒包？

Counterfeit coins I

1 from bag 1

2 from bag 2

...

10 from bag 10

If no counterfeit = $\sum_{i=1}^n 10 \cdot i = 550$

If counterfeit = $550 \pm i$

原题地址: Page 19

Q16: Glass balls 玻璃珠

You are holding two glass balls in a 100-story building. If a ball is thrown out of the window, it will not break if the floor number is less than X , and it will always break if the floor number is equal to or greater than X . You would like to determine X . What is the strategy that will minimize the number of drops for the worst case scenario?¹⁰

你在一座 100 层的大楼里拿着两个玻璃球。
如果一个球被扔出窗外，如果楼层数小于 x ，它不会破裂，如果楼层数等于或大于 x ，它总是会破裂。你想确定 x 。
在最坏的情况下最小化丢弃次数的策略是什么？

Glass Balls

Solution=

First ball = N -th $\begin{cases} \text{breaks} = \text{test from } 1 \text{ to } N-1 \\ \text{No break} = N-1 \text{ throw left} \end{cases}$

Second ball = $(N+N-1)$ $\begin{cases} \text{breaks} = \text{test from } N+1 \text{ to } 2N-2 \\ \text{No breaks} = N-2 \text{ throw left} \end{cases}$

$$N+(N-1)+(N-2)+\dots+1 = \frac{N(N+1)}{2} \geq 100$$

原题地址: Page 19

Q17: Matching socks 袜子匹配

Your drawer contains 2 red socks, 20 yellow socks and 31 blue socks. Being a busy and absent-minded MIT student, you just randomly grab a number of socks out of the draw and try to find a matching pair. Assume each sock has equal probability of being selected, what is the minimum number of socks you need to grab in order to guarantee a pair of socks of the same color?

Matching Socks

Pigeon hole principle

鸽笼理论

Solution=

n Holes, $n+1$ pigeons = at least 2 pigeons share 1 hole

n Holes, $mn+1$ pigeons = at least $m+1$ pigeons share 1

hole

3 colors, \Rightarrow 4 socks

你的抽屉里有 2 只红袜子、20 只黄袜子和 31 只蓝袜子。作为一名忙碌而心不在焉的麻省理工学院学生，您只是随机从抽奖中抓取一些袜子并尝试找到匹配的一双。

假设每只袜子被选中的概率均等，为了保证有一双颜色相同的袜子，最少需要抓取多少只袜子？

原题地址: Page 21

Q18: Prisoner problem 监狱问题

One hundred prisoners are given the chance to be set free tomorrow. They are all told that each will be given a red or blue hat to wear. Each prisoner can see everyone else's hat but not his own. The hat colors are assigned randomly and once the hats are placed on top of each prisoner's head they cannot communicate with one another in any form, or else they are immediately executed. The prisoners will be called out in random order and the prisoner called out will guess the color of his hat. Each prisoner declares the color of his hat so that everyone else can hear it. If a prisoner guesses correctly the color of his hat, he is set free immediately; otherwise he is executed.

They are given the night to come up with a strategy among themselves to save as many prisoners as possible. What is the best strategy they can adopt and how many prisoners can they guarantee to save?¹⁴

一百名囚犯明天有机会获释。他们都被告知，每个人都会得到一顶红色或蓝色的帽子。每个囚犯都可以看到其他人的帽子，但看不到自己的帽子。帽子的颜色是随机分配的，一旦帽子被放在每个囚犯的头顶上，他们就不能以任何形式相互交流，否则他们会被立即处决。囚犯将以随机顺序被叫出，被叫出的囚犯将猜测他帽子的颜色。每个囚犯都宣布自己帽子的颜色，以便其他人都能听到。如果一个囚犯猜对了帽子的颜色，他就会立即被释放；否则他将被处决。

他们被安排在一夜之间想出一个策略来拯救尽可能多的囚犯。他们能采取的最佳策略是什么？他们能保证拯救多少囚犯？

原题地址：Page 24

Prisoners Problem

Solution=

99个人可以活

第一个人：奇数个红帽子，则说红色

偶数个红帽子，则说蓝色

1/2概率=活命

第i个人 剩余98个人 第一个人

Q19: Correlation 相关系数

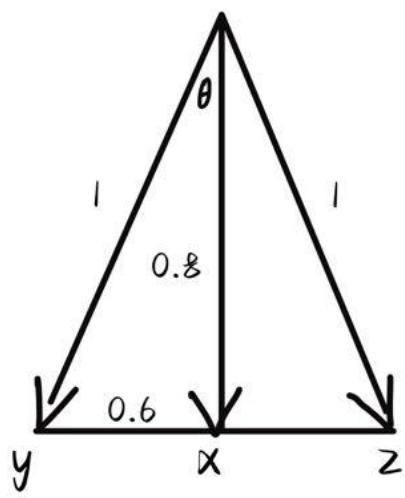
There are 3 random variables x , y and z . The correlation between x and y is 0.8 and the correlation between x and z is 0.8. What is the maximum and minimum correlation between y and z ?

有3个随机变量 x 、 y 、 z , x 与 y 的相关系数为0.8, x 与 z 的相关系数为0.8。
 y 和 z 之间最大和最小的相关性是什么?

原题地址: Page 51

Correlation

解法1



$$\begin{aligned} \cos \theta &= 0.8 \\ \cos 2\theta &= \cos^2 \theta - \sin^2 \theta = 0.8^2 - 0.6^2 = 0.28 \\ \max &= 1 \\ \min &= 0.28 \end{aligned}$$

解法2

$$\rho = \begin{vmatrix} 1 & 0.8 & 1 \\ 0.8 & 1 & \rho \\ 0.8 & \rho & 1 \end{vmatrix}$$

$$\begin{aligned} |\rho| &= (1 - \rho^2) - 0.8(0.8 - 0.8\rho) + 0.8(0.8\rho - 0.8) \\ &= -0.28 + 1.28\rho - \rho^2 \geq 0 \end{aligned}$$

$$\Rightarrow (\rho - 1)(\rho - 0.28) \leq 0$$

$$\Rightarrow 0.28 \leq \rho \leq 1$$

Q20: Normal Generate 正态生成

How do you generate two $N(0,1)$ (standard normal distribution) random variables with correlation ρ if you have a random number generator for standard normal distribution?

如果你有一个标准正态分布的随机数生成器，如何生成两个具有相关性为 ρ 的标准正态分配随机变量？

原题地址: Page 57

Normal generate

$$Z_1 \sim N(0,1) \quad Z_2 \sim N(0,1)$$

$$X_1 = Z_1 \sim N(0,1)$$

$$X_2 = \rho Z_1 + \sqrt{1-\rho^2} Z_2 \sim N(0,1)$$

$$\text{cov}(X_1, X_2) = \text{cov}(Z_1, \rho Z_1 + \sqrt{1-\rho^2} Z_2) = \rho$$

Q21: Coin toss game 抛硬币游戏

Two gamblers are playing a coin toss game. Gambler A has $(n+1)$ fair coins; B has n fair coins. What is the probability that A will have more heads than B if both flip all their coins?²

两个赌徒在玩掷硬币游戏。赌徒A有 $(n+1)$ 枚均匀硬币;B有 n 个均匀硬币。
如果A和B都抛硬币, A正面比B正面多的概率是多少?

Coin toss game

$E1$: A 's n coins have more heads than B 's coins

$E2$: A 's n coins have equal number of heads as
 B 's n coins

$E3$: A 's n coins have less heads than B 's n coins

$$P(E1) = P(E3) = x \quad 2x + y = 1$$

$$P(E2) = y$$

$$\Rightarrow x + 0.5y = 0.5$$

原题地址: Page 61

Q22: Card game 卡片游戏

A casino offers a simple card game. There are 52 cards in a deck with 4 cards for each value 2, 3, 4, 5, 6, 7, 8, 9, 10, jack queen king ace J, Q, K, A. Each time the cards are thoroughly shuffled (so each card has equal probability of being selected). You pick up a card from the deck and the dealer picks another one without replacement. If you have a larger number, you win; if the numbers are equal or yours is smaller, the house wins—as in all other casinos, the house always has better odds of winning. What is your probability of winning?

z Card game

E1= your card number > dealer's

E2= your card number = dealer's

E3= your card number < dealer's

$$P(E1) = P(E3)$$

$$P(E2) = \frac{13 \cdot C_4^2}{C_{52}^2} = \frac{13 \times 4 \times 3}{52 \times 51} = \frac{1}{17}$$

$$\Rightarrow P(E1) = \frac{8}{17}$$

赌场提供简单的纸牌游戏。一副牌中有52张牌，每个杰克女王王牌有4张牌，值2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A。每次彻底洗牌（因此每张牌被选中的概率相等）。你从牌组中取出一张牌，然后经销商在没有更换的情况下再取出一张。如果你有一个更大的数字，你就赢了；如果数字相等或你的数字较小，那么赌场获胜，就像在所有其他赌场一样，赌场获胜的几率总是更高。你获胜的概率是多少？

原题地址: Page 61

Q23: Drink passenger 醉酒乘客

A line of 100 airline passengers are waiting to board a plane. They each hold a ticket to one of the 100 seats on that flight. For convenience, let's say that the n -th passenger in line has a ticket for the seat number n . Being drunk, the first person in line picks a random seat (equally likely for each seat). All of the other passengers are sober, and will go to their proper seats unless it is already occupied; In that case, they will randomly choose a free seat. You're person number 100. What is the probability that you end up in your seat (i.e., seat #100) ?³

100名航空乘客排队等候登机，他们每人持有一张机票。为了方便起见，我们假设排队的第 n 个人有一张号码为 n 的票。由于排队的第一个人喝醉了，他会选择一张随机座位（每个座位的可能性相同），所有其他乘客都很清醒，除非自己座位被占了，否则就会坐到他们本来的座位上；在这种情况下，他们将随机选择一个空座位。如果你是100号人物，问你最终有多大概率坐在自己座位上（即100号座位）？

原题地址：Page 62

Drunk passenger

First person=A

A take # 1 \Rightarrow you will be # 100 ✓

A take # 100 \Rightarrow you can't be #100 ✗

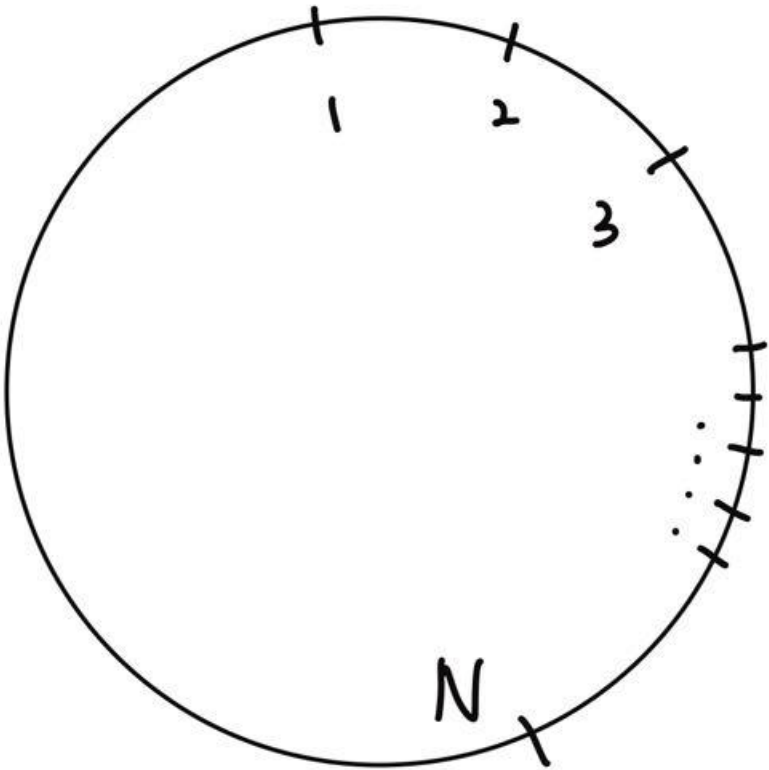
A take # $n \Rightarrow 2 \sim n-1$ would be there seat

$$n \Rightarrow \begin{cases} n \text{ take } \# 1 \text{ you will be } \# 100 \checkmark \\ n \text{ take } \# 100 \text{ you will be } \# 100 \\ \phantom{n \text{ take } \# 100 \text{ you will be } \# 100} \times \end{cases}$$

Q24: N points on a circle 一圆N点

Given N points drawn randomly on the circumference of a circle, what is the probability that they are all within a semicircle?⁴ 给定在圆周上随机绘制的N个点，问他们在同一个半圆内的概率是多少？

N points on a circle



$$P = \frac{N}{2^{N-1}}$$

原题地址: Page 64

Q25: Pocker hands 德州扑克

Poker is a card game in which each player gets a hand of 5 cards. There are 52 cards in a deck. Each card has a value and belongs to a suit. There are 13 values, 2, 3, 4, 5, 6, 7, 8, 9, 10, ^{jack queen king ace} J, Q, K, A, and four suits, ^{spade club heart diamond} ♠, ♣, ♥, ♦.

What are the probabilities of getting hands with four-of-a-kind (four of the five cards with the same value)? Hands with a full house (three cards of one value and two cards of another value)? Hands with two pairs?

扑克是一种纸牌游戏，每个玩家都可以得到一手5张牌。一张牌中有52张卡片，每张卡片都有价值，属于一套。有13个值，2、3、4、5、6、7、8、9、10、J、Q、K、A和四个花色。拿到四张同类牌（五张牌中的四张）的概率是多少具有相同的值？满座的手牌（三张一值牌和两张另一个值）的概率是多少？两对的概率是多少？

原题地址: Page 65

Poker hands

Four-of-a-kind prob= $\frac{C_{13}^1 C_{48}^1}{C_{52}^5}$

Hands of a full house= $\frac{C_{13}^1 C_4^3 C_{12}^1 C_4^2}{C_{52}^5}$

Hands of two pairs= $\frac{C_{13}^2 C_4^2 C_4^2 C_{44}^1}{C_{52}^5}$

Q26: Hopping rabbit 忙碌的兔子

A rabbit sits at the bottom of a staircase with n stairs. The rabbit can hop up only one or two stairs at a time. How many different ways are there for the rabbit to ascend to the top of the stairs?⁵

一只兔子坐在一个有 n 个楼梯的底部。兔子只能一次跳一个或两个楼梯。兔子有多少种不同的方式可以上升到楼梯顶部？

原题地址: Page 66

Hopping rabbit

$$f(1) = 1$$

$$f(2) = 2 \quad 1+1/2$$

$$f(n) = f(n-2) + f(n-1)$$

Q27: Chess tournament 象棋锦标赛

A chess tournament has 2^n players with skills $1 > 2 > \dots > 2^n$. It is organized as a knockout tournament, so that after each round only the winner proceeds to the next round. Except for the final, opponents in each round are drawn at random. Let's also assume that when two players meet in a game, the player with better skills always wins. What's the probability that players 1 and 2 will meet in the final?⁸

国际象棋锦标赛有 2^n 名棋手，排名依次为 $1 > 2 > \dots > 2^n$ 。比赛形式为淘汰赛，因此每一轮之后只有获胜者才能进入下一轮。除了决赛，每一轮的对手都是随机抽取。我们还假设当两个玩家在游戏中相遇时，技能更好的玩家总是获胜。问选手 1 和选手 2 在决赛中相遇的概率是多少？

原题地址: Page 68

Chess tournament

All 2^{n-1} 2^{n-1} $2^n - 1$ 个位置可选

1 vs 2
Finally 2^{n-1} 2^{n-1} 2^{n-1} 个位置可选

$$\frac{2^{n-1}}{2^n - 1}$$

Q28: Birthday problem 生日问题

How many people do we need in a class to make the probability that two people have the same birthday more than $1/2$? (For simplicity, assume 365 days a year.)

一个班级需要多少人才能使至少两个人有相同生日的概率大于 $1/2$?
(为简单起见, 假设一年有 365 天)

Birthday problems

$$n \geq 23$$

n 个人

$$\frac{365 \times 364 \times \dots \times (365 - n + 1)}{365^n} < 1/2$$

原题地址: Page 71

Q29: 100th digit 第100个数字

What is the 100th digit to the right of the decimal point in the decimal representation of $(1 + \sqrt{2})^{3000}$?

$(1 + \sqrt{2})^{3000}$ 的小数点后第100位是多少?

原题地址: Page 71

100th digit

$(1 + \sqrt{2})^{3000} + (1 - \sqrt{2})^{3000}$ 是个整数

$$(1 - \sqrt{2})^{3000} < (0.5)^{3000} = (0.125)^{1000} \leq 10^{-100}$$

\Rightarrow 整数部分为9

Q30: Cubic of integer 整数的三次方

Let x be an integer between 1 and 10^{12} , what is the probability that the cubic of x ends with 11?

设 x 是从1到 10^{12} 之间的整数，问 x 的三次方结尾末两位数字为11的概率是多少？

Cubic of integer

原题地址: Page 72

$$x = (a + 10b)$$

$$x^3 = (a + 10b)^3 = a^3 + 30a^2b + 300ab^2 + 1000b^3$$

$$\begin{cases} a^3 \equiv 1 \\ 3a^2b \equiv 1 \end{cases} \Rightarrow \begin{cases} a \equiv 1 \\ b \equiv 1 \end{cases}$$

$a, b \in [0, 1, \dots, 9]$ Last 2 digits is 11

Q31: Boys and girls 男孩和女孩

Part A. A company is holding a dinner for working mothers with at least one son. Ms. Jackson, a mother with two children, is invited. What is the probability that both children are boys?

Part B. Your new colleague, Ms. Parker is known to have two children. If you see her walking with one of her children and that child is a boy, what is the probability that both children are boys?

Part A: 一家公司正在为至少有一个儿子的职业母亲举办晚宴。有两个孩子的母亲杰克逊女士受邀参加。两个孩子都是男孩的概率是多少？

Part B: 你的新同事帕克女士有两个孩子。如果你看到她和她一个孩子一起散步，而那个孩子是个男孩，那么两个孩子都是男孩的可能性有多大？

原题地址: Page 73

Boys and girls

$$\text{Part A} = \frac{1}{3} \begin{pmatrix} b & g \\ g & b \\ b & b \end{pmatrix}$$

$$\text{Part B} = \frac{1}{2}$$

Q32: All-girl world 女生世界

In a primitive society, every couple prefers to have a baby girl. There is a 50% chance that each child they have is a girl, and the genders of their children are mutually independent. If each couple insists on having more children until they get a girl and once they have a girl they will stop having more children, what will eventually happen to the fraction of girls in this society?

在原始社会，每对夫妇都希望生一个女婴。他们所生的每个孩子都有50%的几率是女孩，而且他们孩子的性别是相互独立的。如果每对夫妇都坚持要多生，直到生女孩，生了女孩就不再生孩子，那么这个社会的女孩比例最终会发生什么变化？

All - girl world

50%

原题地址: Page 74

Q33: Unfair coin 不公平的硬币

You are given 1000 coins. Among them, 1 coin has heads on both sides. The other 999 coins are fair coins. You randomly choose a coin and toss it 10 times. Each time, the coin turns up heads. What is the probability that the coin you choose is the unfair one?

给你 1000 个硬币。其中，1 枚硬币两面都是正面。其他 999 个硬币是普通双面硬币。您随机选择一枚硬币并掷 10 次，每次硬币都会正面朝上。您选择的硬币是不公平硬币的概率是多少？

Unfair coin

原题地址: Page 74

Bayesian

$$P(A/B) = \frac{P(B/A)P(A)}{P(B/A)P(A) + P(B/A^c)P(A^c)} = \frac{1/1000 \times 1}{1/1000 \times 1 + 999/1000 \times \frac{1}{1024}} \approx 0.5$$

Q34: Fair probability from an unfair coin 不公平硬币的公平概率

If you have an unfair coin, which may bias toward either heads or tails at an unknown probability, can you generate even odds using this coin?

如果你有一枚不公平的硬币，它可能以未知的概率偏向正面或反面，你能用这枚硬币产生成败机会相等的赔率吗？

Fair probability

$$P(H) \neq P(T)$$

HH TH HT TT

$$P(TH) = P(HT)$$

原题地址: Page 75

Q35: Monty Hall problem 蒙蒂霍尔问题

Monty Hall problem is a probability puzzle based on an old American show *Let's Make a Deal*. The problem is named after the show's host. Suppose you're on the show now, and you're given the choice of 3 doors. Behind one door is a car; behind the other two, goats. You don't know ahead of time what is behind each of the doors.

You pick one of the doors and announce it. As soon as you pick the door, Monty opens one of the other two doors that he knows has a goat behind it. Then he gives you the option to either keep your original choice or switch to the third door. Should you switch? What is the probability of winning a car if you switch?

蒙蒂霍尔问题是一个基于美国老节目 *Let's Make a Deal* 的概率谜题。这个问题以节目主持人的名字命名。假设你现在在节目中，你可以选择 3 扇门。一扇门后面是一辆汽车；在另外两个后面是山羊。你无法提前知道每一扇门后面是什么。你选择其中一扇门并宣布它。一旦你选择了门，蒙蒂就会打开另外两扇门中的一扇（他知道这扇门后面有一只山羊）。然后他给你选择，要么保留你原来的选择，要么切换到第三扇门。你应该换吗？如果换的话，赢得汽车的概率是多少？

原题地址：Page 78

Monty Hall problem

$$\boxed{\frac{1}{3}} \quad \frac{1}{3} \quad \frac{1}{3}$$

$$\frac{1}{3} \quad \boxed{0 \quad \frac{2}{3}}$$

Q36: Amoeba population 变形虫种群

There is a one amoeba in a pond. After every minute the amoeba may die, stay the same, split into two or split into three with equal probability. All its offspring, if it has any, will behave the same (and independent of other amoebas). What is the probability the amoeba population will die out?

池塘里有一只变形虫。每一分钟后，变形虫可能死亡，保持不变，分裂成两个或分裂成三个概率相等。它的所有后代也会有类似的表现（并且独立于其他变形虫）。

问变形虫种群灭绝的概率是多少？

Amoeba population

$$P(E) = 1/4 + \frac{1}{4} \times P(E) + \frac{1}{4} \times P(E)^2 + \frac{1}{4} (P(E))^3$$
$$\Rightarrow P(E) = \sqrt{2} - 1$$

原题地址: Page 79

Q37: Coin toss game 抛硬币游戏

Two players, A and B , alternatively toss a fair coin (A tosses the coin first, then B tosses the coin, then A , then B ...). The sequence of heads and tails is recorded. If there is a head followed by a tail (HT subsequence), the game ends and the person who tosses the tail wins. What is the probability that A wins the game?¹⁷

两个玩家A和B，交替投掷一枚公平的硬币（A先投掷硬币，然后B投掷硬币，然后是A，然后是B...）。记录硬币头部和尾部的出现顺序，如果有一个头跟着一个尾（HT子序列），游戏结束，抛尾的人获胜。A赢得比赛的概率是多少？

Coin toss game

原题地址: Page 80

$P(A)$ = the probability that A wins

$P(B)$ = _____ B _____

$$P(A) = 1/2 P(A/H) + 1/2 P(A/T)$$

$$P(A/T) = P(B) = 1 - P(A)$$

$$P(A/H) = 1/2 \times 0 + 1/2 \times (1 - P(A/H)) \Rightarrow P(A/H) = 1/3$$

$$P(A) = 1/2 \times 1/3 + 1/2(1 - P(A)) \Rightarrow P(A) = 4/9$$

Q38: Probability of triangle 三角形概率

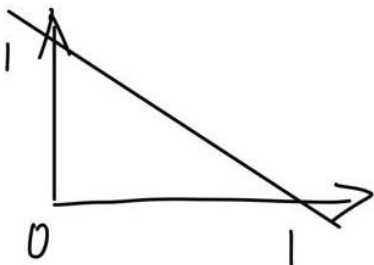
A stick is cut twice randomly (each cut point follows a uniform distribution on the stick), what is the probability that the 3 segments can form a triangle?²² 一根木棍被随机切割两次（每个切割点在木棍上服从均匀分布），问这3段可以形成三角形的概率是多少？

Probability of triangle

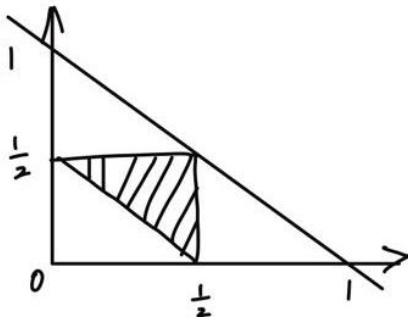
原题地址: Page 89

x y $1-x-y$

$$\begin{cases} x > 0 \\ y > 0 \\ x-y < 0 \end{cases}$$



$$\begin{cases} x+y > 1-x-y \\ 1-y > y \\ 1-x > x \end{cases} \Rightarrow \begin{cases} x+y > \frac{1}{2} \\ y < \frac{1}{2} \\ x < \frac{1}{2} \end{cases}$$



$$\frac{1}{4}$$

Q39: Connection noodles 连接面

You have 100 noodles in your soup bowl. Being blindfolded, you are told to take two ends of some noodles (each end on any noodle has the same probability of being chosen) in your bowl and connect them. You continue until there are no free ends. The number of loops formed by the noodles this way is stochastic. Calculate the expected number of circles.

汤碗里有 100 条面条。你被蒙住眼睛后，随意挑选一些面条的两端（任何面条的每一端被选中的概率相同）放入碗中并将它们连接起来，直到没有自由的端结束。问最终形成圈数的期望。

Connection noodles

原题地址: Page 93

$$n=1 \quad E(f(1)) = 1$$

$$n=2 \quad E(f(2)) = 2/6 \times (1+E(f(1))) + 4/6 E(f(1))$$

$$C_4^2 = 6 \quad = 1/3 + E(f(1)) = 1/3 + 1$$

$$n=3 \quad E(f(3)) = 3/15 (1+E(f(2))) + 12/15 \times E(f(2))$$

$$C_6^2 = 15 \quad = 1/5 + E(f(2)) = 1/5 + 1/3 + 1$$

$$n=4 \quad E(f(4)) = \frac{4}{28} (1+E(f(3))) + \frac{24}{28} \times E(f(3))$$

$$C_8^2 = 28 \quad = 1/7 + 1/5 + 1/3 + 1$$

Q40: Optimal hedge ratio 对冲比例

You just bought one share of stock A and want to hedge it by shorting stock B . How many shares of B should you short to minimize the variance of the hedged position? Assume that the variance of stock A 's return is σ_A^2 ; the variance of B 's return is σ_B^2 ; their correlation coefficient is ρ .

您刚买了一股股票 A 并想通过卖空股票 B 来对冲它。您应该卖多少 B 股才能使对冲头寸的方差最小? 假设股票 A 的回报方差是 σ_A^2 , B 的回报的方差是 σ_B^2 , 他们的相关系数是 ρ

Optimal hedge ratio

原题地址: Page 94

$$\begin{aligned} \text{Var}(r_A - h r_B) \\ &= \sigma_A^2 - 2\rho\sigma_A\sigma_B h + h^2\sigma_B^2 \\ \frac{\partial \text{Var}}{\partial h} &= -2\rho\sigma_A\sigma_B + 2h\sigma_B^2 \\ \Rightarrow h &= \rho \frac{\sigma_A}{\sigma_B} \end{aligned}$$

Q41: Card game 扑克牌游戏

What is the expected number of cards that need to be turned over in a regular 52-card deck in order to see the first ace? 在常规 52 张牌中，为了看到第一张 A，需要翻开的牌张数的期望是多少？

Card game

原题地址: Page 95

4 Aces 48 other cards labeled 1, 2, ..., 48

$$X_i = \begin{cases} 1 \\ 0 \end{cases} \quad \text{If card } i \text{ turn over before 4 aces}$$

1 A 2 A 3 A 4 A 5

$$E(X_i) = 1/5$$

$$X = 1 + \sum_{i=1}^{48} X_i \quad E(X) = 1 + \sum_{i=1}^{48} E(X_i)$$

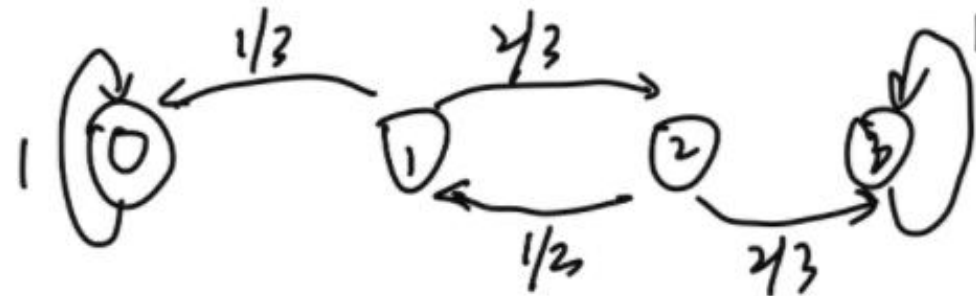
$$E(X) = 1 + \frac{48}{5} = 10.6$$

Q42: Gambler's ruin problem 赌徒问题

Player M has \$1 and player N has \$2. Each game gives the winner \$1 from the other. As a better player, M wins $2/3$ of the games. They play until one of them is bankrupt. What is the probability that M wins?

玩家M有1美元，玩家N有2美元。每局给获胜者1美元。M每次赢得比赛的概率是 $2/3$ ，他们会一直玩，直到其中一个破产。问M赢的概率是多少？

Gambler's ruin problem



$$a_1 = \frac{1}{3} \times 0 + \frac{2}{3} \times a_2$$

$$a_2 = \frac{1}{3} \times a_1 + \frac{2}{3} \times 1$$

$$\Rightarrow a_1 = 4/7 \quad a_2 = 6/7$$

So, starting from \$1, M has $4/7$ probability of winning.

原题地址: Page 107

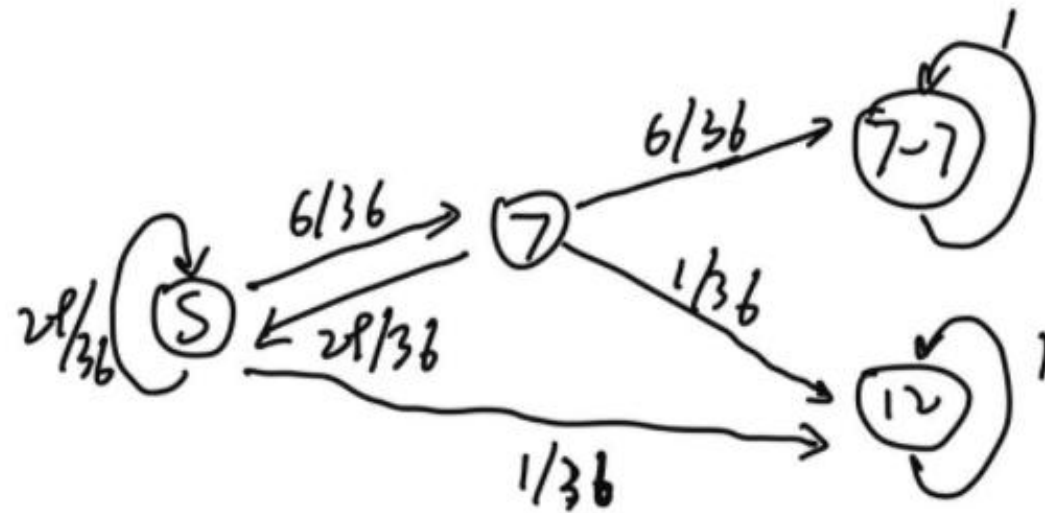
Q43: Dice question 筛子问题

Two players bet on roll(s) of the total of two standard six-face dice. Player A bets that a sum of 12 will occur first. Player B bets that two consecutive 7s will occur first. The players keep rolling the dice and record the sums until one player wins. What is the probability that A will win?

两名玩家通过两个标准六面骰子下注，玩家A打赌和为12将首先出现，玩家B打赌先出现两个连续的7。两位玩家不断地掷骰子并记录总数，直到一名玩家获胜。问A获胜的概率是多少？

Dice question

原题地址: Page 108



$$\begin{aligned} a_{12} &= 1 & a_{7-7} &= 0 \\ a_5 &= 1/36 \times 1 + 6/36 \times a_7 + 28/36 \times a_5 \\ a_7 &= 1/36 \times 1 + 6/36 \times 0 + 28/36 \times a_5 \end{aligned}$$

$$\Rightarrow a_5 = 7/13 \quad a_7 = 6/13$$

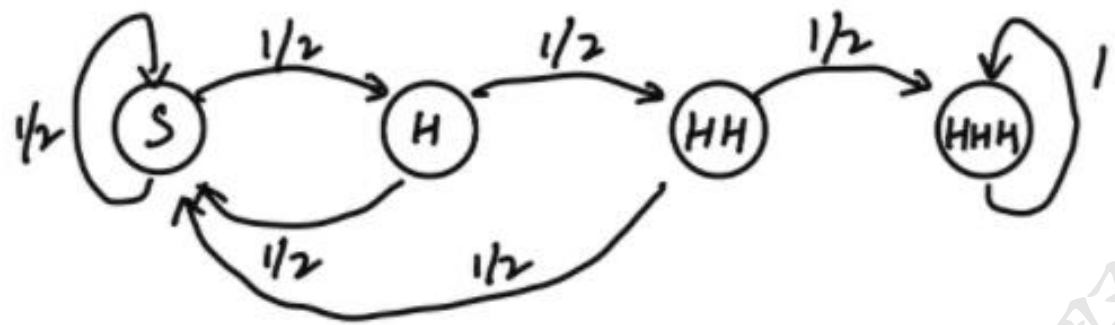
Q44: Coin triplets 硬币三连抛

Part A. If you keep on tossing a fair coin, what is the expected number of tosses such that you can have HHH (heads heads heads) in a row? What is the expected number of tosses to have THH (tails heads heads) in a row?

如果你继续扔一个公平的硬币，那么预期的投掷多少次，可以连续出现三个头 HHH ？预期的投掷多少次，可以出现 THH ？

原题地址：Page 109

Coin triplets



$$\begin{aligned} \mu_S &= 1 + \frac{1}{2} \mu_S + \frac{1}{2} \mu_H \\ \mu_H &= 1 + \frac{1}{2} \mu_S + \frac{1}{2} \mu_{HH} \\ \mu_{HH} &= 1 + \frac{1}{2} \mu_S + \frac{1}{2} \mu_{HHH} \\ \mu_{HHH} &= 0 \end{aligned} \Rightarrow \begin{aligned} \mu_S &= 14 \\ \mu_H &= 12 \\ \mu_{HH} &= 8 \\ \mu_{HHH} &= 0 \end{aligned}$$

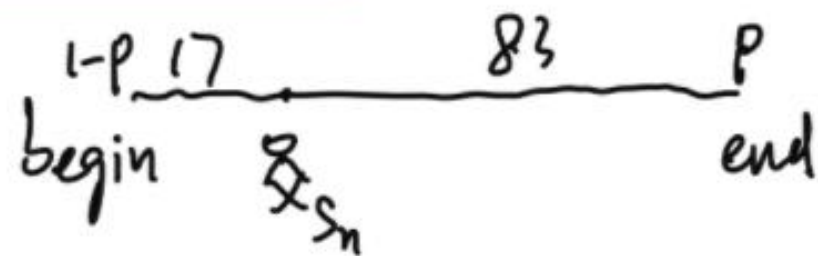
Q45: Drunk Man 醉汉

A drunk man is at the 17th meter of a 100-meter-long bridge. He has a 50% probability of staggering forward or backward one meter each step. What is the probability that he will make it to the end of the bridge (the 100th meter) before the beginning (the 0th meter)? What is the expected number of steps he takes to reach either the beginning or the end of the bridge?

一个醉汉在一座100米长的桥的17米处。他有50%的可能性一步向前或向后错开一米。他最终能够在不到达起点（第0米）的情况下，先到达大桥的终点（第100米）的概率是多少？他预计期望要走多少步才能到达桥的两端？

原题地址: Page 116

Drunk man



$$S_n = 83 \text{ or } -17 \text{ step}$$

$$S_n, S_n^2 - n \text{ 鞅}$$

$$E(S_n) = p \times 83 + (1-p) \times (-17) = 0 \quad p = 0.17$$

$$\begin{aligned} E(S_n^2 - n) &= E(p \times 83^2 + (1-p) \times 17^2) - E(n) \\ &= E(S_0^2) - 0^2 = 0 \end{aligned}$$

$$\Rightarrow E(n) = 1441$$

Q46: Dice game 筛子游戏

Suppose that you roll a dice. For each roll, you are paid the face value. If a roll gives 4, 5 or 6, you can roll the dice again. If you get 1, 2 or 3, the game stops. What is the expected payoff of this game?

假设你掷骰子。对于每一次投掷，您将获得其面值。如果一个滚动给出4，5或者6，你可以再次掷骰子。如果你得到1，2或3，游戏停止。问这个游戏的预期收益是多少？

原题地址：Page 117

Dice game

stop probability $p=1/2$

$$E(N) = 1/p = 2$$

$$E(x) = (1+2+\dots+6)/6 = \frac{7}{2}$$

$$E(S_n) = E(N) \cdot E(x) = 2 \cdot \frac{7}{2} = 7$$

Q47: Coin sequences 硬币序列

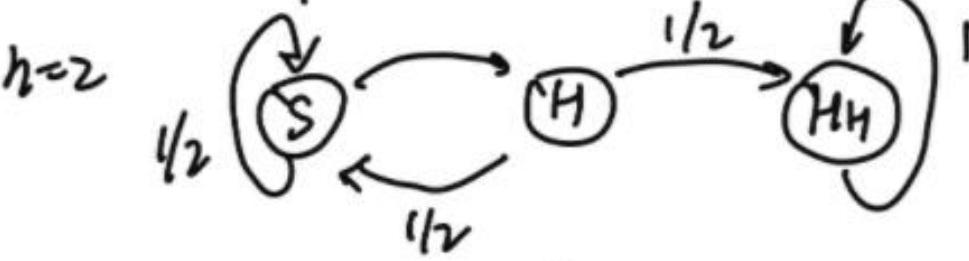
Assume that you have a fair coin. What is the expected number of coin tosses to get n heads in a row?

假设你有一枚公平的硬币，预计投掷硬币使得出现连续N个头的期望次数？

原题地址: Page 119

Coin sequence

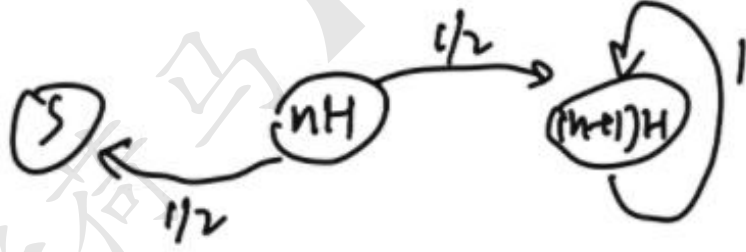
$n=1 \quad E(f(1)) = 2$



$$\begin{aligned} \mu_S &= 1 + \frac{1}{2}\mu_S + \frac{1}{2}\mu_H \\ \mu_H &= 1 + \frac{1}{2}\mu_S \\ \mu_{HH} &= 0 \end{aligned} \Rightarrow \begin{aligned} \mu_S &= 6 \\ \mu_H &= 4 \\ \mu_{HH} &= 0 \end{aligned}$$

$n=3 \quad E(f(3)) = 14 \quad Q44$

guess $E(f(n)) = 2^{n+1} - 2$



$$\begin{aligned} E(f(n+1)) &= E(f(n)) + 1 + \frac{1}{2}E(f(n+1)) \\ \Rightarrow E(f(n+1)) &= 2E(f(n)) + 2 = 2^{n+2} - 2 \end{aligned}$$

Q48: Dice Game 筛子游戏

You can roll a 6-side dice up to 3 times. After the first or the second roll, if you get a number x , you can decide either to get x dollars or to choose to continue rolling. But once you decide to continue, you forgo the number you just rolled. If you get to the third roll, you'll just get x dollars if the third number is x and the game stops. What is the game worth and what is your strategy?

你可以掷六面骰子最多3次。在第一轮或第二轮之后，如果你得到数字 x ，您可以决定要么获得 x 美元，要么选择继续滚动。但是一旦你决定继续，你就放弃了刚才滚动的数字。如果你到了第三个滚，如果第三个数字是 x ，游戏停止，你只会得到 x 美元。请问这个游戏的期望价值和策略是什么？

Dice game

1st round expectation $\frac{7}{2}$

4, 5, 6 stop

1, 2, 3 go on

$$E(x) = \frac{3}{6} \times 3.5 + \frac{1}{6} (4+5+6) = 4.25$$

2nd round expectation 4.25

5, 6 stop

1, 2, 3, 4 go on

$$E(x) = \frac{4}{6} \times 4.25 + \frac{1}{6} \times (5+6) = 4.66$$

原题地址: Page 123

Q49: Brownian motion 布朗运动I

What is the correlation of a Brownian motion and its square?

布朗运动与其平方的相关系数是什么？

原题地址: Page 130

Brownian motion I

$$\begin{aligned} B_t &\sim N(0, t) \\ \text{cov}(B_t, B_t^2) &= E(B_t^3) - E(B_t) E(B_t^2) \\ &= 0 - 0 \cdot t = 0 \end{aligned}$$

Q50: Brownian motion 布朗运动 II

Let B_t be a Brownian motion. What is the probability that $B_1 > 0$ and $B_2 < 0$? 设 B_t 为布朗运动, 请问 $B_1 > 0$ 和 $B_2 < 0$ 的概率是多少?

Brownian motion II

原题地址: Page 130

$$B_1 \sim N(0, 1) \quad B_2 - B_1 \sim N(0, 1)$$

$$B_1 > 0, B_2 < 0 \Leftrightarrow B_1 > 0, B_2 - B_1 < -B_1$$

$$\Leftrightarrow B_1 > 0, B_2 - B_1 < 0$$

$$|B_2 - B_1| > |B_1|$$

$$P(B_1 > 0, B_2 < 0) = P(B_1 > 0) P(B_2 - B_1 < 0)$$

$$P(|B_2 - B_1| > |B_1|)$$

$$= 1/2 \cdot 1/2 \cdot 1/2 = 1/8$$

Q51: Arbitrage Opportunity 套利机会

A European put option on a non-dividend paying stock with strike price \$80 is currently priced at \$8 and a put option on the same stock with strike price \$90 is priced at \$9. Is there an arbitrage opportunity existing in these two options?

执行价格为 80 美元的无股息支付股票的欧式看跌期权定价为 8 美元，同一股票的执行价格为 90 美元的看跌期权定价为 9 美元。这两个期权是否存在套利机会？

Arbitrage Opportunity

原题地址: Page 141

short 1 units of put with $K=80$
long 2 units of put with $K=90$

$$S \geq 90 \quad P_{NL} = 0$$

$$90 > S \geq 80 \quad P_{NL} = 8 \times (90 - S) > 0$$

$$\begin{aligned} S < 80 \quad P_{NL} &= 8 \times (90 - S) - 1 \times (80 - S) \\ &= 72 - 7S > 0 \end{aligned}$$

Q52: Black-Scholes BS模型

What are the assumptions behind the Black-Scholes formula?

Black-Scholes公式背后的假设是什么？

原题地址: Page 144

➤ **市场有效性假设:**

- 该模型假设市场是完全有效的，即没有摩擦或交易成本，并且所有信息是充分的和即时的。这意味着没有利用市场中的套利机会。

➤ **连续性假设:**

- 该模型假设证券价格的变化是连续的，并且价格变动是无限小的。这意味着证券价格的随机行为可以通过几何布朗运动 (Geometric Brownian Motion) 来建模。

➤ **对数正态分布假设:**

- 该模型假设证券价格的对数收益率是正态分布的。这意味着证券价格的百分比变化是对数正态分布的，并且这一假设是通过观察市场数据得出的。

➤ **无风险利率恒定假设:**

- 该模型假设无风险利率是一个已知的恒定值，并且在期权的整个期限内不变。

➤ **无交易费用和税费假设:**

- 该模型假设在交易期间没有交易费用、税费或其他摩擦成本。

➤ **可以自由买卖标的资产假设:**

- 该模型假设在任何时间都可以自由地买入或卖出期权的标的资产。

Q53: Portfolio optimization 组合优化

You are constructing a simple portfolio using two stocks *A* and *B*. Both have the same expected return of 12%. The standard deviation of *A*'s return is 20% and the standard deviation of *B*'s return is 30%; the correlation of their returns is 50%. How will you allocate your investment between these two stocks to minimize the risk of your portfolio?

您正在使用两只股票 **A** 和 **B** 构建一个简单的投资组合。两者具有相同的预期回报12%。**A**的回报率标准差为20%，**B**的回报标准差为30%；它们的回报率相关性为50%。你将会怎样在这两只股票之间分配您的投资，以最大限度地降低您的投资组合的风险？

原题地址: Page 164

Portfolio optimization

$$\begin{aligned}\text{Var}(r_p) &= \sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2\rho_{AB}\sigma_A\sigma_B w_A w_B \\ &= \sigma_A^2 w_A^2 + \sigma_B^2 (1-w_A)^2 + \\ &\quad 2\rho_{AB}\sigma_A\sigma_B w_A (1-w_A)\end{aligned}$$

$$\frac{\partial \text{Var}(r_p)}{\partial w_A} = 0 \Rightarrow$$

$$w_A = \frac{\sigma_B^2 - \rho_{AB}\sigma_A\sigma_B}{\sigma_A^2 - 2\rho_{AB}\sigma_A\sigma_B + \sigma_B^2} = \frac{6}{7}$$

Q54: Value at risk 在险值

Briefly explain what VaR is. What is the potential drawback of using VaR to measure the risk of derivatives? 简单解释一下什么是VaR？使用 VaR 来衡量衍生品的风险的潜在缺点是什么？

VaR 定义:

原题地址: Page 164

VaR (Value at Risk) 是一种用于衡量金融投资组合或资产风险的指标。它代表了在一定置信水平下，投资组合或资产在未来一定时间内可能遭受的最大损失额。

潜在缺点:

假设的风险分布: VaR的计算通常基于一些假设，如正态分布假设或其他特定的概率分布假设。然而，在现实中，金融市场的价格变动可能不满足这些假设，导致VaR无法准确预测风险。

缺乏考虑尾部风险: VaR只关注在给定置信水平下的预期最大损失，而忽视了尾部风险（即极端事件发生的风险）。在金融市场中，极端事件的发生可能对投资组合或资产造成巨大的损失，而VaR未能完全捕捉到这种风险。

置信水平解释的不清晰性: VaR提供了一个在给定置信水平下的数值，但对于一般投资者来说，这个数值可能难以理解和解释。同时，不同的置信水平会导致不同的VaR值，这增加了对VaR解释的复杂性。

时间尺度的限制: VaR通常基于一个特定的时间段进行计算，但它无法提供关于风险在不同时间尺度上的演化情况。因此，VaR无法全面反映长期投资或持有期间的风险。

Q55: Number Swap 数据交换

How do you swap two integers, i and j , without using additional storage space?

如何在不用额外存储空间的情况下交换两个整数 i 和 j ?

Number Swap

原题地址: Page 172

$$\begin{aligned} 1) \quad & i = i + j \\ & j = i - j \Rightarrow j = (i + j) - j = i \\ & i = i - j \Rightarrow i = (i + j) - i = j \end{aligned}$$

$$\begin{aligned} 2) \quad & \text{XOR} \quad x \wedge x = 0 \quad x \wedge 0 = x \\ & i = i \wedge j \\ & j = j \wedge i \Rightarrow j = j \wedge (i \wedge j) = i \\ & i = i \wedge j \Rightarrow i = (i \wedge j) \wedge i = j \end{aligned}$$

Q56: Horner's algorithm 多项式求和

Write an algorithm to compute $y = A_0 + A_1x + A_2x^2 + A_3x^3 + \cdots + A_nx^n$.

用最快速的方法计算 $y = A_0 + A_1x + A_2x^2 + A_3x^3 + \cdots + A_nx^n$.

Horner's algorithm

原题地址: Page 174

$$\begin{aligned} y &= A_0 + A_1x + \cdots + A_nx^n \\ &= (((A_nx + A_{n-1})x + A_{n-2})x + \cdots + A_2)x \\ &\quad + A_1)x + A_0 \end{aligned}$$

$$\wedge B_n = A_n \quad B_{n-1} = B_nx + A_{n-1}$$

$$B_0 = B_1x + A_0$$

Q57: Moving Average 移动平均

Given a large array A of length m , can you develop an efficient algorithm to build another array containing the n -element moving average of the original array ($B_1, \dots, B_{n-1} = NA, B_i = (A_{i-n+1} + A_{i-n+2} + \dots + A_i) / n, \forall i = n, \dots, m$)?

给定一个长度为 m 的数组 A ，你能开发一种高效的算法计算移动平均数吗？

Moving average

$$O(n^2) \Rightarrow O(n)$$

$$S_{1,n} = A_1 + \dots + A_n$$

$$S_{2,n+1} = S_{1,n} + A_{n+1} - A_1$$

$$S_{m,n+m-1} = S_{m-1,n+m-2} + A_{n+m-1} - A_{n-1}$$

原题地址: Page 174

Q58: Sorting Algorithm I 搜索算法I

Develop an algorithm to find both the minimum and the maximum of n numbers using no more than $3n/2$ comparisons.

开发一种算法来查找 n 个数字的最小值和最大值，使用不超过 $3n/2$ 次比较。

Search algorithm I

原题地址: Page 177

$$n-1 + n-1 = 2n-2$$

① $n/2$ pair max group
 min group

② min group $n/2-1$

③ max group $n/2-1$

$$\frac{3n}{2} - 2$$

Q59: Sorting Algorithm II 搜索算法 II

You are given an array of numbers. From the beginning of the array to some position, all elements are zero; after that position, all elements are nonzero. If you don't know the size of the array, how do you find the position of the first nonzero element?

给你一个数字数组，从数组的开头到某个位置，所有元素均为零；在该位置之后，所有元素均非零。如果您不知道数组的大小，如何找到第一个非零元素的位置？

原题地址: Page 178

Search algorithm II

$$1, 2, 4, \dots, 2^i, 2^{i+1}, \dots, 2^n \dots$$

$$\text{if } X_{2^i} = 0 \quad X_{2^{i+1}} > 0$$

$$\text{mid} = \frac{2^i + 2^{i+1}}{2}$$

$$2^i \sim \frac{2^i + 2^{i+1}}{2}$$

$$\frac{2^i + 2^{i+1}}{2} \sim 2^{i+1}$$

Q60: Maximum contiguous subarray 最大连续子数组

Suppose you have a one-dimensional array A with length n that contains both positive and negative numbers. Design an algorithm to find the maximum sum of any contiguous subarray $A[i, j]$ of A : $V(i, j) = \sum_{x=i}^j A[x], 1 \leq i \leq j \leq n$.

假设有一个长度为 n 的一维数组 A ，其中包含正数和负数，设计一个算法来找到任何连续的最大求和子数组。

原题地址: Page 180

Maximum contiguous subarray

for $i=1 \dots n$

for $j=1 \dots n$

$O(n^2)$

动态规划

$$S_n = \max(S_{n-1} + a_n, a_n)$$